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Analysis of combined electroosmotic and pressure-driven flow in cylindrical microchannels considering interfacial slip effects and slip-dependent zeta potential

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Extended Abstract

Introduction

Electrokinetic transport phenomena in microchannels have attracted considerable attention because of their important applications in microfluidic systems, electroosmotic micropumps, lab-on-a-chip devices, micro-mixers, micro-separators, and drug-delivery systems. In these systems, electroosmotic flow and pressure-driven flow are among the most common mechanisms for controlling and transporting fluids. Although the no-slip boundary condition is frequently employed in conventional fluid mechanics, it may not adequately describe fluid behavior at solid–liquid interfaces in microscale systems. In particular, hydrodynamic slip can significantly modify the velocity field and, consequently, the transport characteristics of electroosmotic flows. The interaction between wall slip and electrokinetic effects is also associated with changes in the zeta potential. Since the actual surface potential is difficult to measure directly, the zeta potential is generally used to characterize the electrokinetic behavior of the solid–liquid interface. Previous investigations have considered electroosmotic and pressure-driven flows under different geometries and boundary conditions. However, most existing studies have either assumed a no-slip boundary condition, treated the zeta potential as independent of the slip length, or investigated only limited geometries such as parallel-plate microchannels. Therefore, the combined effect of wall slip and slip-dependent zeta potential in cylindrical microchannels remains insufficiently explored. The main objective of the present study is to analytically investigate fully developed combined electroosmotic and pressure-driven flow of an aqueous electrolyte in a cylindrical microchannel while considering wall slip and the dependence of zeta potential on slip length. The effects of the Debye parameter, slip parameter, pressure-gradient parameter, and zeta potential on the electric potential distribution, velocity profile, wall shear stress, skin-friction coefficient, and volumetric flow rate are systematically examined. The principal novelty of the study is the simultaneous consideration of slip-dependent zeta potential and hydrodynamic slip in a cylindrical microchannel and the derivation of analytical expressions for the principal flow characteristics.

2. Materials and Methods

The physical model consists of a cylindrical microchannel containing an aqueous electrolyte solution. An external electric field is imposed along the axial direction of the channel, generating electroosmotic motion through the interaction between the electric field and the ions within the electrical double layer. In addition, a uniform axial pressure gradient is applied to generate pressure-driven flow. Depending on its direction, the pressure-driven component can either enhance or oppose the electroosmotic flow.

The flow is assumed to be steady, incompressible, Newtonian, fully developed, axisymmetric, and one-dimensional. The thermophysical properties of the fluid are assumed to remain constant over the considered temperature range. The electrolyte is modeled as an ideal aqueous symmetric electrolyte. The Debye–Hückel linearization is employed to describe the electrical double layer, and streaming-potential effects are neglected because their magnitude is assumed to be small compared with the externally imposed electric potential. The electrical potential distribution is obtained from the Poisson–Boltzmann equation. Under the Debye–Hückel approximation, this equation is linearized and solved analytically in cylindrical coordinates. The effect of wall slip is incorporated into the boundary condition at the solid–liquid interface. Since wall slip modifies ion adsorption and the surface charge distribution, the zeta potential is not considered constant; instead, it is modeled as a function of the slip length and electrolyte properties based on previously established relationships. The axial momentum equation is then formulated by incorporating both the pressure-gradient force and the electroosmotic body force. An analytical solution is derived for the dimensionless velocity profile. Based on the obtained velocity field, the wall shear stress, skin-friction coefficient, and volumetric flow rate are calculated. The governing analytical formulation is evaluated over a broad range of the principal dimensionless parameters, including the Debye parameter, slip parameter, zeta potential, and pressure-gradient parameter.The validity of the analytical formulation is verified through several comparisons. The dimensionless electric-potential distribution under no-slip conditions is compared with previously published analytical results, while the velocity profile in the absence of electrokinetic effects is compared with the classical Hagen–Poiseuille solution. Additional comparisons are performed for pure electroosmotic flow and the net charge-density distribution in an aqueous KCl solution. The close agreement between the present analytical predictions and the available reference solutions confirms the validity of the developed model.

3. Results and Discussion

The results demonstrate that the Debye parameter, wall-slip parameter, zeta potential, and pressure gradient have substantial effects on the electrokinetic behavior of the flow. Increasing the Debye parameter corresponds to a reduction in the thickness of the electrical double layer. Under these conditions, the electrical double layer occupies only a small region adjacent to the microchannel wall, while a larger portion of the channel remains outside the double layer. An increase in the slip parameter produces a considerable enhancement in the electric potential and modifies the thickness of the electrical double layer. A similar trend is observed when the wall zeta potential increases. Because the zeta potential is explicitly modeled as slip-dependent, the electric potential at the surface deviates from the nominal zeta potential as the slip length increases. These findings demonstrate that ignoring the coupling between wall slip and zeta potential may lead to an inaccurate description of electrokinetic transport in microchannels. The pressure gradient determines the relative contribution of pressure-driven and electroosmotic transport. When the pressure gradient acts in the same direction as electroosmotic flow, the flow is strengthened, and the maximum velocity occurs near the center of the cylindrical microchannel. In contrast, an opposing pressure gradient weakens the overall flow and shifts the location of the maximum velocity toward the wall. For sufficiently strong adverse pressure gradients, a reverse-flow region develops in the central portion of the microchannel. Thus, an opposing pressure gradient can substantially alter the velocity distribution and may eventually produce a negative net volumetric flow rate.

The slip parameter has one of the most pronounced effects on the velocity field. Within the investigated range, increasing the dimensionless slip parameter from 0 to approximately 0.1 increases the maximum velocity by nearly seven times. This result highlights the strong influence of hydrodynamic slip on microscale electroosmotic transport. Increasing the zeta potential also enhances the velocity, although its influence is somewhat different because the zeta potential affects the flow indirectly through the slip-dependent electrokinetic boundary condition. The effect of the zeta potential becomes particularly evident away from the wall, while its influence on the velocity gradient near the wall can be even more pronounced. The volumetric flow rate increases when the pressure gradient acts in the direction of electroosmotic flow and decreases when the pressure gradient opposes it. Increasing the zeta potential strengthens the electroosmotic contribution and consequently increases the volumetric flow rate. A more significant enhancement is observed with increasing wall slip, particularly when the electrical double layer is thin and the magnitude of the zeta potential is relatively high. The volumetric flow rate varies linearly with the dimensionless pressure-gradient parameter for fixed values of the other governing parameters. Furthermore, reversal of the zeta-potential sign changes the direction of the electroosmotic contribution because the sign of the accumulated surface charge and the orientation

of the mobile ions in the diffuse layer are reversed. The skin-friction coefficient is also strongly influenced by the thickness of the electrical double layer. For relatively thick electrical double layers, the friction coefficient approaches an approximately constant value. As the double layer becomes thinner, however, the wall velocity gradient becomes much steeper and the skin-friction coefficient increases substantially. An adverse pressure gradient can reduce the wall shear stress and, under certain conditions, can even reverse its sign. Increasing the slip length and zeta potential generally increases the magnitude of the skin-friction coefficient. The dependence of the friction coefficient on the pressure-gradient parameter is approximately linear. Moreover, the friction coefficient varies linearly with the slip parameter, while the slope of this relationship depends on the zeta potential and Debye parameter. From an engineering perspective, the results indicate that hydrodynamic slip and electrokinetic surface properties can be used as effective control parameters for microscale fluid transport. Increasing wall slip and controlling the zeta potential can enhance the volumetric flow rate and modify frictional losses, thereby providing opportunities for improving the energy efficiency and controllability of electrokinetic microfluidic systems.

4. Conclusions

An analytical investigation was conducted to characterize fully developed combined electroosmotic and pressure-driven flow of an aqueous electrolyte in a cylindrical microchannel while accounting for hydrodynamic wall slip and slip-dependent zeta potential. Analytical expressions were derived for the dimensionless electric potential, velocity profile, volumetric flow rate, and skin-friction coefficient. The analytical model was validated against previously reported solutions and the classical Hagen–Poiseuille flow, showing excellent agreement. The results demonstrate that wall slip and zeta potential play critical roles in controlling the electrokinetic characteristics of the flow. Increasing the slip parameter significantly enhances the flow velocity, with the maximum velocity increasing by approximately seven times over the investigated range. An increase in zeta potential similarly strengthens the electroosmotic contribution and increases the volumetric flow rate. A pressure gradient acting in the same direction as electroosmotic flow enhances the flow, whereas an opposing pressure gradient reduces it and may lead to the formation of a reverse-flow region in the central part of the microchannel. The volumetric flow rate varies linearly with the applied pressure-gradient parameter and increases significantly for larger slip lengths, higher zeta potentials, and thinner electrical double layers. The skin-friction coefficient also depends strongly on the electrokinetic parameters and tends toward a constant value for sufficiently thick electrical double layers. In the absence of electrokinetic effects, both volumetric flow rate and friction behavior become independent of the slip and electrical-double-layer parameters. Overall, the findings demonstrate that the coupled effects of wall slip and slip-dependent zeta potential should be considered when modeling and designing cylindrical microfluidic systems. The analytical results provide useful guidance for controlling flow rate, velocity distribution, and frictional losses in electroosmotic micropumps, lab-on-a-chip systems, drug-delivery devices, and electrokinetic separation systems.

Key words: Electroosmotic flow, Microchannel, Interfacial slip, Slip-dependent zeta potential, Analytical solution

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